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Modeling of Quadcopter UAVS has Six Degrees of Freedom

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Abstract

Quadcopter, also known as quadrotor or drone, belongs to a particular type of Vertical Take-Off and Landing aircraft with four directed upward rotors. The electric motors and their corresponding propellers are usually placed in a square formation with an equal distance to the center of mass. Similar to the other types of multi-rotor drone, quadrotor is controlled by adjusting angular velocities of the propellers. Quadcopters have generated considerable interest in both the control community due to their complex dynamics and a lot of potentials in outdoor applications because of their advantages over regular aerial vehicles.

This work addresses the modelling for quadcopter or drone unmanned aerial vehicles (UAVs). the mathematical model of the drone is derived by identifying significant parameters and the negligible ones are treated as disturbances.

Keywords: Quadcopter, Drone, Nonlinear Model

Introduction

In research, the quadcopter is an exemplary design for small unmanned aerial vehicles with six degrees of freedom but only four independent inputseds. To gain the six degrees of freedom, rotational and translational motions are coupled. Under the circum-stances, dynamics of this flying object are highly nonlinear, particularly beneath the effect of the aerodynamics. Besides, quadrotor has microscopic friction to prevent its movement, so it must yield its own damping to block the move and remain in a steady state.

The quadrotor or quadcopter are unmanned aerial vehicles capable of vertical take-off and landing (VTOL). Maneuverability is high. Although control systems are complex, they are structurally simple. It has four rotors and the rotors are positioned equal distance from the quadrotor center of mass. They utilize the forces produced by the rotors and are unmanned aerial vehicles with rotating wing, which form the thrust force by means of propellers. They differ from the standard helicopters in using rotors with fixed-pitch blades. Quadrotors have many advantages over standard helicopters or manned aircraft. Some of these advantages; low production costs, the ability to add features according to need and eliminate the risk of the pilots in hazardous work environments [1]. In particular, quadrotors have been used in many areas, including hazardous and dangerous areas where people cannot dissolve. In civilian use, quadrotors are being used in areas such as hobby, agriculture, aerial photography and firefighting. In military use, quadrotors are used in many areas such as determination of enemy forces, port and coast security, land search, surveillance, mine screening, long distance and high-altitude discoveries, spy communication, determination of radar systems [2]. It has attracted the attention of many researchers due to its success in search and rescue, exploration and security [3-9]. The development of unmanned aerial vehicles (UAV) has generated great interest in the automatic control area in the last few decades. Throughout the history the UAV has been an invisible player in military applications [10-12] and civilian applications [13-16], usually used for surveillance, border patrolling, mine detection, aerial delivery of payload, forest fires monitoring, environmental protection, film production, etc.

The unmanned aerial robot quadrotor full control and modeling was working on. His mathematical model was nonlinear and benefited from Newtonian laws of motion. He used the state variables approach in the control system and made the simulations by creating the model [17]. In [18, 19] the design, the quadrotor was controlled via wireless from the ground control center. PID was used as the control. The results showed that the quadrotor showed stable attitude with PID control and compensated under disturbance. Yogianandh, Riaan and Glen [20], they conducted a study on the quadrotor dynamic model. They used PD for quadrotor control and simulated matlab / simulink. A quadrotor using the PID controller worked on attitude control [21-25] by turning parameter of PID controller. Matlab / Simulink made simulations and concluded that the recommended controller provided adequate performance. As the technology advancement makes smaller yet smarter electronic components and actuators possible. A relatively new type of rotorcraft, the so-called quadrotor, has slowly became the main focus in realizing the concepts due to its simplicity.

Mathematical models of Quadcopter

Similar to any rotorcrafts, quadrotor has the potential to takeoff, hover, fly, and land, but in a much constrained and smaller area. It has a simple control mechanism, where the main actuators are the four independent propellers attached at each corner of the aircraft body. For a standard quadrotor, it consists of a pair of propellers rotating in the counter clockwise direction (direction for standard propellers) and another pair in the reverse direction to produce zero net yaw torque.

The quadcopter model is shown in the Fig 1. Its dynamics is set up by two coordinate systems, namely earth frame (inertial frame) and the body fixed frame (body frame). The inertial frame (x_E, y_E, z_E) is defined by the ground, with z_E pointing down to the earth centre. The body frame (x_B, y_B, z_B) is specified by the orientation of the quadcopter, with the rotor axes pointing downward and the arms pointing in the x_B and y_B directions.

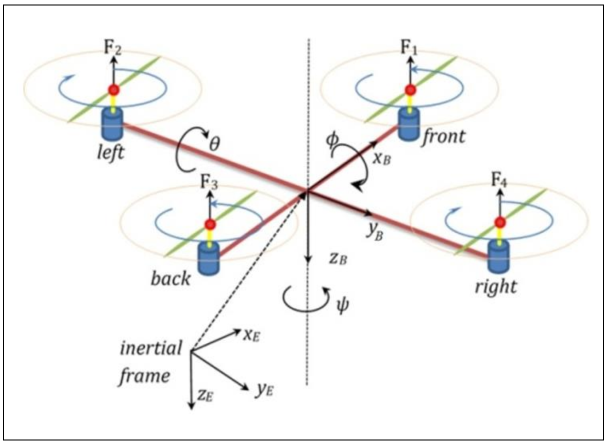

Fig 1: Quadcopter structure and coordinate systems

The equations representing the motion of the quadcopter are basically those of a rotating rigid body with six degrees of freedom, i.e., three translational and three rotational motions. The translational movements are defined in the earth frame, where the position is presented in vector form as $\xi = (x, y, z)^T$ and the vector $\dot{\xi} = (\dot{x}, \dot{y}, \dot{z})^T$ denotes its linear velocity. The drone attitude is defined by using the three Euler angles, named roll, pitch, and yaw are determined in the body frame as $\Theta = (\varphi, \theta, \psi)T$, their corresponding angular rates are performed as $\Theta^{\cdot} = (\varphi^{\cdot}, \theta^{\cdot}, \psi^{\cdot})T$.

Let $\omega = [p, q, r]T$ represents the angular rate vector in the inertial frame. Then, the following rotational kinematics is achieved to show the relation between the earth angular velocity and the Euler angle rate vectors:

$$\omega = \begin{bmatrix} 1 & 0 & -s_\theta \\ 0 & c_\phi & c_\theta s_\phi \\ 0 & -s_\phi & c_\theta c_\phi \end{bmatrix} \tag{1}$$

where $sx = sin(x)$, $cx = cos(x)$.

The below transformation matrix defines the relation between the body frame to earth frame translational velocities:

$$R = \begin{bmatrix} c_\psi c_\theta & c_\psi s_\theta c_\phi - s_\psi c_\phi & c_\psi s_\theta c_\phi + s_\psi s_\phi \\ s_\psi c_\theta & s_\psi s_\theta s_\phi + c_\psi c_\phi & s_\psi s_\theta c_\phi - c_\psi s_\phi \\ s_\theta & c_\psi s_\theta & c_\theta c_\phi \end{bmatrix} \tag{2}$$

Since the ultimate objective of this study is the attitude control, only torque elements that are capable to vary the quadcopter orientation are taken into account. They include torques caused by thrust forces τ, body gyroscopic effects τ_b, propeller gyroscopic effects τp, and aerodynamic friction τ_a.

The torque τ is produced by the quadcopter according to the body frame. τ consists of roll, pitch and yaw torque components, i.e., $\tau = [\tau_\varphi\ \tau_\theta\ \tau_\psi]T$. They are performed as:

$$\tau_\phi = l(F_2 - F_4) \tag{3}$$

$$\tau_\theta = l(-F_1 + F_3) \tag{4}$$

$$\tau_\psi = c(-F_1 + F_2 - F_3 + F_4) \tag{5}$$

where F_k, k = 1, 2, 3, 4, is the thrust force generated by the propeller k, l is the distance from a motor to the drone center of mass and c is a force-to-torque scaling coefficient.

The body gyroscopic torque is modelled as:

$$\tau_b = S(\omega) I \omega \tag{6}$$

where $S(\omega)$ is a skew-symmetric matrix for the given vector ω, and is expressed as follows:

$$S(\omega) = \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \tag{7}$$

τ_p is the resultant of torques generated by propeller gyroscopic effects, τ_p is determined as:

$$\tau_p = \begin{bmatrix} I_r \Omega_r q \\ -I_r \Omega_r p \\ 0 \end{bmatrix} \tag{8}$$

where I_r is the inertial moment of rotor, $\Omega_r = -\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4$ is the residual angular velocity of rotor in which Ω_k denotes the angular velocity of the propeller k. The aerodynamic friction torque τ_a is given by:

$$\tau_a = k_a \omega^2 \tag{9}$$

where ka is a diagonal positive definite matrix of aerodynamic friction coefficients, ka= diag[k_{ax}, k_{ay}, k_{az}].

Using the aforementioned torques, the overall attitude dynamic model of the quadcopter is derived as:

$$I\ddot{\Theta} = \tau_b + \tau + \tau_p - \tau_a \tag{10}$$

where I is a diagonal positive definite matrix of inertia tensors when the quadrotor is assumed to be symmetrical, $I = diag[I_{xx}, I_{yy}, I_{zz}]$.

In our study, the gyroscopic and aerodynamic torques are considered as external disturbances, and they are supposed t_o be removed by the advancement of the proposed controller. Therefore, the control inputs mainly depend on torque τ and from (3), (4) and (5), they can be represented as:

$$\begin{bmatrix} u_\phi \\ u_\theta \\ u_\psi \\ u_z \end{bmatrix} = \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \\ F \end{bmatrix} = \begin{bmatrix} 0 & l & 0 & -l \\ -l & 0 & l & 0 \\ -c & c & -c & c \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} \tag{11}$$

where u_ϕ, u_θ and u_ψ respectively represent the roll, pitch and yaw torques, u_z represents the total thrust acting on the four rotors and F denotes the UAV lift produced by the four propellers, $F = \sum_{i=1}^{k} F_k$. In this paper, u_z is supposed to accommodate with the gravity when we consider the rotational control only. In view of the equations from (3) to (7), the second-order nonlinear dynamics of quadcopters for attitude control can be described by the following equations:

$$\ddot{\phi} = \frac{I_{yy} - I_{zz}}{I_{xx}} qr + \frac{1}{I_{xx}} u_\phi + \frac{1}{I_{xx}} d_\phi \tag{12}$$

$$\ddot{\theta} = \frac{I_{zz} - I_{xx}}{I_{yy}} pr + \frac{1}{I_{yy}} u_\theta + \frac{1}{I_{yy}} d_\theta \tag{13}$$

$$\ddot{\psi} = \frac{I_{xx} - I_{yy}}{I_{zz}} pq + \frac{1}{I_{zz}} u_\psi + \frac{1}{I_{zz}} d_\psi \tag{14}$$

where $d\varphi$, $d\theta$ and $d\psi$ are the disturbances, including the two terms τa in (8), τp in (9) to the system in real-time.
Let us define the following state variables:

$$\begin{aligned} X_1 &= \Theta \\ X_2 &= \dot{\Theta} \end{aligned} \tag{15}$$

Then, the dynamics of quadcopters can be represented in the following form as:

$$\begin{cases} \dot{X}_1 = X_2 \\ \dot{X}_2 = I^{-1}\left[f(X) + u(t) + d(t) \right] \end{cases} \tag{16}$$

where $u = [u_\varphi, u_\theta, u_\psi]T$ is the input vector, and $d = [d_\varphi, d_\theta, d_\psi]T$ is the disturbance vector, the vector $f(X)$ is represented as:

$$f(X) = -S(\omega)I\omega = \begin{bmatrix} (I_{yy} - I_{zz})qr \\ (I_{zz} - I_{xx})pr \\ (I_{xx} - I_{yy})pq \end{bmatrix} \tag{17}$$

Conclusion

Knowledge of the dynamics of the quadrotor is essential when designing a controller. Quadcopters have generated considerable interest in both the control community due to their complex dynamics and much potential in outdoor applications because of their advantages over regular aerial vehicles. Future research will focus on machine learning and deep learning techniques that play a promising role in different applications related to UAVs, such as battery scheduling, trajectory planning, tracking, obstacle avoidance, and resource allocation. The development of new ML tools and enhancement of onboard computational power will help to develop novel UAV models that are smarter, lightweight, and smaller to perform any operation without the risk of collision. Using these tools, UAVs can autonomously modify their motion, direction, and location to serve ground users. Moreover, the availability of accurate data can support UAVs in intelligent control, trajectory planning, and vision tasks.

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