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Application of Computer Optimization Methods in the Reconstruction of Railway Lines

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Abstract

The main task that railway transport faces in the near future is to increase the speed of trains and reduce travel time on existing railway lines using computer technology and mathematical modeling methods. The problem of determining the optimal speeds of trains of each category under optimum outer rail cant, compliance with the condition of uniform load on rail tracks, in which the running time of trains along a curve of length will be minimal has been formulated. The problem is considered when train speeds are limited by the locomotive traction force and are determined on the basis of traction calculations or natural measurements. As a problem-solving technique, indefinite Lagrange multipliers method and its applicability in algebra system MathCAD have been proposed.

Keywords: Computer Optimization Methods, Speed, Outstanding Acceleration

1. Introduction

The current state of the theory of railway transport is characterized by the presence of a system of mathematical models and algorithms for analyzing various features of railway lines. This makes it possible to build an approach to the problems of search and optimization of design solutions on a mathematical basis. This will reduce expensive and lengthy physical modeling procedures [1, 2, 3].

One of the main tasks that railway transport faces in the near future is to increase the speed of trains and reduce travel time on existing railway lines using computer technology (computer-aided design) [4]. The use of computer optimization methods makes it possible to ensure the entire decision-making process at all stages of the development of a railway line reconstruction project for high speeds. The implementation of such support takes place through the organization of management of information sources, selection of the best characteristics of mathematical models and effective management of the design process [6, 7].

The need for modeling the design process of reconstruction of railway lines appears in order to provide a specialist in the field of system analysis with means of describing the design technology being developed.

2. Materials and methods

The problem of determining the optimal speeds of train of each category in curves is formulated as follows:

Assume, there is a railway stretch with m independent (single-radius or compound) curves on it and running trains of j-category $(j = \overline{1,n})$.

For each category of trains are known:

Q_j – the mass of trains of the j-category (t),
N_j – number of trains of j-category,
a_j – outstanding acceleration (m / s^2).

It is necessary to determine speeds v_j (km / h) of j-category trains $(j = \overline{1,n})$ on a curve of certain radius R (m) at optimum outer rail cant h (mm), compliance with the condition of uniform load on rail tracks h (mm), in which the running time of trains along a curve of length l (m) will be minimal:

$$T = f(v_1, v_2, \ldots, v_n) = l\sum_{j=1}^{n}\frac{N_j}{v_j} \rightarrow min \quad (1)$$

in restricting: compliance with the condition of uniform load on rail tracks:

$$v_{av}^2 = \sum_{j=1}^{n} c_j v_j^2 = 3,6^2\frac{ghR}{S}, \quad (2)$$

Where

v_{av}^2 – average square speed (km / h);

g = 9,81 m / s^2 – gravity acceleration;

S – track width between the longitudinal axes of the rails (m);

$c_j = \frac{N_j Q_j}{\sum_{j=1}^{n} N_j Q_j}$ – present value of j-category train.

To solve the problem (1) – (2) it was applied indefinite Lagrange multipliers method [10, 11] and the following results were obtained.

Let us compose Lagrange function:

$$L(v_1, v_2, \ldots, v_n, \lambda) = l\sum_{j=1}^{n}\frac{N_j}{v_j} + \lambda\left(3,6^2\frac{ghR}{S} - \sum_{j=1}^{n} c_j v_j^2\right) \quad (3)$$

Then identify partial derivative of Lagrange function for unknowns v_j $(j = \overline{1,n})$, λ, and set them to zero. As a result, we obtain the following system of equations:

$$\frac{\partial L(v_1, \ldots, v_n, \lambda)}{\partial v_j} = -l\frac{N_j}{v_j^2} + 2\lambda c_j v_j = 0, (j = \overline{1,n}); \quad (4)$$

$$\frac{\partial L(v_1, \ldots, v_n, \lambda)}{\partial \lambda} = 3,6^2\frac{ghR}{S} - \sum_{j=1}^{n} c_j v_j^2 = 0. \quad (5)$$

As follows from the expression (4):

$$lN_j = 2\lambda c_j v_j^3, \quad (6)$$

where from

$$v_j = \left(\frac{lN_j}{2\lambda c_j}\right)^{\frac{1}{3}}. \quad (7)$$

Substitute (7) into (5) and find $\frac{1}{2\lambda}$:

$$\sum_{j=1}^{n} c_j\left(\frac{lN_j}{2\lambda c_j}\right)^{\frac{2}{3}} = 3,6^2\frac{ghR}{S}, \quad (8)$$

$$\sum_{j=1}^{n}\left(\frac{lN_j}{2\lambda}\right)^{\frac{2}{3}} c_j^{\frac{1}{3}} = 3,6^2\frac{ghR}{S}, \quad (9)$$

$$\left(\frac{1}{2\lambda}\right)^{\frac{2}{3}} = \frac{3,6^2 ghR}{S\sum_{j=1}^{n}(lN_j)^{\frac{2}{3}} c_j^{\frac{1}{3}}}, \quad (10)$$

$$\frac{1}{2\lambda} = \left(\frac{3,6^2 ghR}{S\sum_{j=1}^{n}(lN_j)^{\frac{2}{3}} c_j^{\frac{1}{3}}}\right)^{\frac{3}{2}}, \quad (11)$$

Next, replacing $\frac{1}{2\lambda}$ in (7), we get the speeds of j-category trains.

The optimal speeds of j-category trains in curves of certain radius at optimal rail cant, compliance with the condition of uniform load on rail tracks are the following:

$$v_j = \left(\frac{3{,}6^2 ghR}{S\sum_{j=1}^{n} c_j^{\frac{1}{3}} N_j^{\frac{2}{3}}}\right)^{\frac{1}{2}} \left(\frac{N_j}{c_j}\right)^{\frac{1}{3}}, (j = \overline{1,n}). \tag{12}$$

Taking into account the optimal speeds of *j*-category trains in curve of certain radius, the amount of travel time will be minimal and of:

$$T = l\sum_{j=1}^{n} N_j \left[\left(\frac{3{,}6^2 ghR}{S\sum_{j=1}^{n} c_j^{\frac{1}{3}} N_j^{\frac{2}{3}}}\right)^{\frac{1}{2}} \left(\frac{N_j}{c_j}\right)^{\frac{1}{3}}\right]^{-1} \tag{13}$$

Unbalanced acceleration of *j*-category train is determined by the formula:

$$a_j = \frac{v_j^2}{3{,}6^2 R} - \frac{gh}{S}, (j = \overline{1,n}), \tag{14}$$

or taking into account (12)

$$a_j = \frac{gh}{S}\left(\frac{\left(\frac{N_j}{c_j}\right)^{\frac{2}{3}}}{\sum_{j=1}^{n} c_j^{\frac{1}{3}} N_j^{\frac{2}{3}}} - 1\right), (j = \overline{1,n}) \tag{15}$$

As follows from (15), unbalanced acceleration a_j does not depend on radius of curve *R*.
In real conditions, train speeds are limited by the locomotive traction force and are determined on the basis of traction calculations or natural measurements.
Denote by v_j^f (km / h) the speed set on the basis of traction calculations of *j*-category trains $(j = \overline{1,q})$, and through v_j, the speed determined through condition (2) $(j = \overline{1,p})$, and $p + q = n$. Then task (1)-(2) can be written as

$$T = f(v_1, v_2, \ldots, v_p, v_1^f, v_2^f, \ldots, v_q^f) = l\left(\sum_{j=1}^{p}\frac{N_j}{v_j} + \sum_{j=1}^{q}\frac{N_j}{v_j^f}\right) \to min \tag{16}$$

$$v_{av}^2 = \sum_{j=1}^{p} c_j v_j^2 + \sum_{j=1}^{q} c_j \left(v_j^f\right)^2 = 3{,}6^2\frac{ghR}{S} \tag{17}$$

In order to solve the problem of interest (16) – (17), it is appropriate to apply indefinite Lagrange multipliers method [10, 11].

Let us compose Lagrange function:

$$L(v_1, v_2, \ldots, v_p, v_1^f, v_2^f, \ldots, v_q^f, \lambda) = l\left(\sum_{j=1}^{p}\frac{N_j}{v_j} + \sum_{j=1}^{q}\frac{N_j}{v_j^f}\right) + \lambda\left(3{,}6^2\frac{ghR}{S} - \sum_{j=1}^{p} c_j v_j^2 - \sum_{j=1}^{q} c_j \left(v_j^f\right)^2\right) \tag{18}$$

where λ – indefinite Lagrange multiplier.

The optimal speeds of *j*-category trains in curves of certain radius at optimal rail cant, compliance with the condition of uniform load on rail tracks are the following:

$$v_j = \left(\frac{\frac{3{,}6^2 ghR}{S} - \sum_{j=1}^{q} c_j v_j^f}{\sum_{j=1}^{p} c_j^{\frac{1}{3}} N_j^{\frac{2}{3}}}\right)^{\frac{1}{2}} \left(\frac{N_j}{c_j}\right)^{\frac{1}{3}}, j = \overline{1,p}, j = \overline{1,q} \tag{19}$$

At the same time, there should be

$$h \succ S\left(\sum_{j=1}^{q} c_j \left(v_j^f\right)^2\right) / (3{,}6^2 gR).$$

Taking into account the optimal speeds of *j*-category trains (19) in curve of certain radius, unbalanced acceleration of *j*-category train from (14) is determined by the formula:

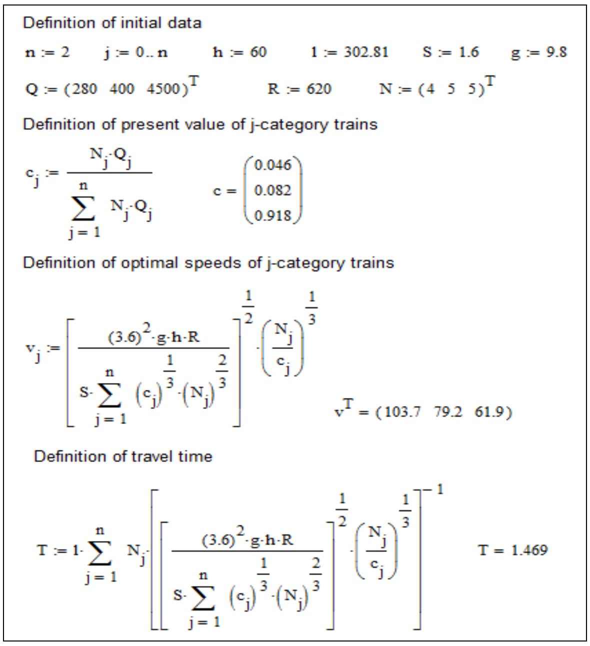
$$a_j = \frac{g}{S}\left[\frac{\left(\frac{N_j}{c_j}\right)^{\frac{2}{3}}}{\sum_{j=1}^{p} c_j^{\frac{1}{3}} N_j^{\frac{2}{3}}} - 1\right]h - \frac{1}{3{,}6^2 R}\frac{\sum_{j=1}^{q} c_j \left(v_j^f\right)^2}{\sum_{j=1}^{p} c_j^{\frac{1}{3}} v_j^{\frac{2}{3}}}\left(\frac{N_j}{c_j}\right)^{\frac{2}{3}}, j = \overline{1,p} \quad (20)$$

The optimal solution of problem (16)-(17) is represented by formulas (19)-(20) on the upper and right border of the rectangle $a_j \leq a_{ad}$ and $h_j \leq h_{ad}$.

3. Research results

Quantitative solution to the problem of determining the optimal speeds of *j*-category trains in curves of certain radius at optimal rail cant, compliance with the condition of uniform load on rail tracks using Lagrange multipliers method, will be considered on the example of Belarusian Railways section on the Gomel – Minsk direction in Zhlobin district and Gomel district.

Using computer optimization in algebra system MathCAD the values of optimal speeds of *j*-category trains and the amount of travel time in Zhlobin district and Gomel district are obtained (Figures 1-2).


Definition of initial data

$n := 2 \quad j := 0..n \quad h := 60 \quad l := 302.81 \quad S := 1.6 \quad g := 9.8$

$Q := (280 \ \ 400 \ \ 4500)^T \quad R := 620 \quad N := (4 \ \ 5 \ \ 5)^T$

Definition of present value of j-category trains

$c_j := \frac{N_j \cdot Q_j}{\sum_{j=1}^{n} N_j \cdot Q_j} \qquad c = \begin{pmatrix} 0.046 \\ 0.082 \\ 0.918 \end{pmatrix}$

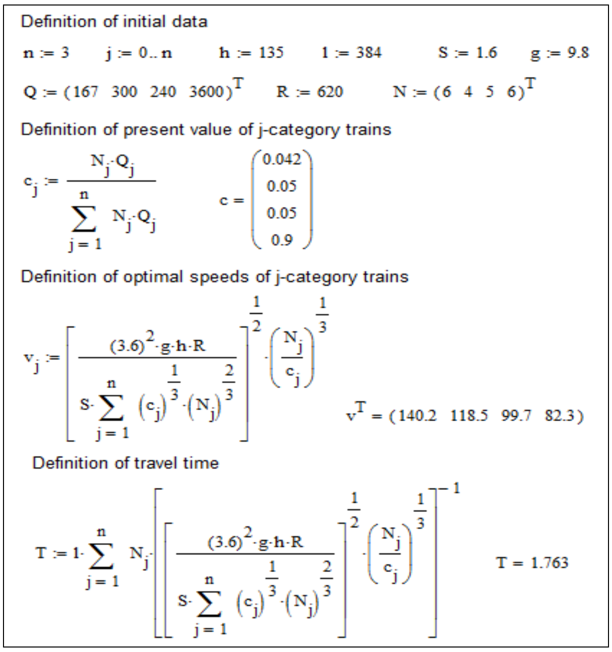
Definition of optimal speeds of j-category trains

$v_j := \left[\frac{(3.6)^2 \cdot g \cdot h \cdot R}{S \cdot \sum_{j=1}^{n} (c_j)^{\frac{1}{3}} \cdot (N_j)^{\frac{2}{3}}}\right]^{\frac{1}{2}} \cdot \left(\frac{N_j}{c_j}\right)^{\frac{1}{3}} \qquad v^T = (103.7 \ \ 79.2 \ \ 61.9)$

Definition of travel time

$T := l \cdot \sum_{j=1}^{n} N_j \cdot \left[\left[\frac{(3.6)^2 \cdot g \cdot h \cdot R}{S \cdot \sum_{j=1}^{n} (c_j)^{\frac{1}{3}} \cdot (N_j)^{\frac{2}{3}}}\right]^{\frac{1}{2}} \cdot \left(\frac{N_j}{c_j}\right)^{\frac{1}{3}}\right]^{-1} \qquad T = 1.469$

Fig 1: Quantitative solution to the problem using computer algebra system MathCAD for Belarusian Railways section on the Gomel – Minsk direction in Zhlobin district


Definition of initial data

$n := 3 \quad j := 0..n \quad h := 135 \quad l := 384 \quad S := 1.6 \quad g := 9.8$

$Q := (167 \ \ 300 \ \ 240 \ \ 3600)^T \quad R := 620 \quad N := (6 \ \ 4 \ \ 5 \ \ 6)^T$

Definition of present value of j-category trains

$c_j := \frac{N_j \cdot Q_j}{\sum_{j=1}^{n} N_j \cdot Q_j} \qquad c = \begin{pmatrix} 0.042 \\ 0.05 \\ 0.05 \\ 0.9 \end{pmatrix}$

Definition of optimal speeds of j-category trains

$v_j := \left[\frac{(3.6)^2 \cdot g \cdot h \cdot R}{S \cdot \sum_{j=1}^{n} (c_j)^{\frac{1}{3}} \cdot (N_j)^{\frac{2}{3}}}\right]^{\frac{1}{2}} \cdot \left(\frac{N_j}{c_j}\right)^{\frac{1}{3}} \qquad v^T = (140.2 \ \ 118.5 \ \ 99.7 \ \ 82.3)$

Definition of travel time

$T := l \cdot \sum_{j=1}^{n} N_j \cdot \left[\left[\frac{(3.6)^2 \cdot g \cdot h \cdot R}{S \cdot \sum_{j=1}^{n} (c_j)^{\frac{1}{3}} \cdot (N_j)^{\frac{2}{3}}}\right]^{\frac{1}{2}} \cdot \left(\frac{N_j}{c_j}\right)^{\frac{1}{3}}\right]^{-1} \qquad T = 1.763$

Fig 2: Quantitative solution to the problem using computer algebra system MathCAD for Belarusian Railways section on the Gomel – Minsk direction in Gomel district

4. Conclusions

The problem reviewed above for each line stretch make it possible to determine the optimal speeds of trains of various categories in curves of certain radius at optimal rail cant, compliance with the condition of uniform load on rail tracks, the amount of travel time and unbalanced acceleration for each category train.

Application of computer optimization methods in reconstruction of express and high-speed railroad lines will make it possible to search for the optimal solutions in the event of certain statement of the problem without significant material expenses that at this time are one of the most important criteria of each research.

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