Int. j. adv. multidisc. res. stud. 2023; 3(1):1011-1013

Received: 29-12-2022
Accepted: 09-02-2023

International Journal of Advanced Multidisciplinary Research and Studies

ISSN: 2583-049X

Modified Ratio Estimator in Survey Sampling using a linear combination

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Abstract

A new modified ratio estimator of population mean of the main variable by means of the linear combination of known values of Co-efficient of Kurtosis and Tri-Mean of the auxiliary variable has been worked out. Mean Square Error (MSE) and Bias of the proposed estimator is calculated and compared with Yan & Tian (2010) estimator. The comparison is demonstrated theoretically and illustrated practically and the conclusion is drawn that the proposed estimator is performing deftly with the above existing estimator.

Keywords: Estimator, Auxiliary variable, Mean square error, Bias, Co-efficient of Kurtosis, Tri-Mean

1. Introduction

The information on main variable Y is studied with the information on secondary variable X also known as auxiliary variable. This information on auxiliary variable X, may be better employed to obtain a more efficient estimator of the population mean. The method is utilized under the condition only if the variables are correlated and do impact on one another. when the population parameters of the auxiliary variable X such as Population Mean, Tri-mean, Coefficient of variation, Kurtosis, Skewness, Correlation coefficient, Median etc., are known, a number of estimators such as ratio, product and linear regression estimators and their modifications are available in literature and are performing better than the simple random sample mean under certain conditions. Among these estimators the ratio estimator and its modifications have widely attracted researchers throughout the world for the estimation of the mean of the study variable (see for example (Kadilar and Cingi, 2004, 2006a, b; Cingi and Kadilar, 2009; Subramani, 2013, Subzar *et al* (2018a, b); Subzar *et al* 2019; Khare and Khare 2019; Priya and Tailor 2019; Neha and Pachori 2019; Priyaranjan and Sunani 2019) [3, 1, 12, 13, 15, 6, 10, 9, 11]. Further, we know that the values of linear combinations are less affected by the extreme values or presence of outliers in the population values. These facts have motivated the authors to propose modified ratio estimators using the above linear combinations. The basic ratio estimator for population mean $\bar{Y}$ is defined as:

$$\hat{\bar{Y}}_R = \frac{\bar{y}}{\bar{x}}\bar{X} = \hat{R}\bar{X}, \qquad (1.1)$$

Where $\hat{R} = \frac{\bar{y}}{\bar{x}} = \frac{y}{x}$ is the estimate of $R = \frac{\bar{Y}}{\bar{X}} = \frac{Y}{X}$ and it is assumed that the population mean $\bar{X}$ of the auxiliary variate X is known. Hence $\bar{y}$ is the sample mean of the variate of the interest and $\bar{x}$ is the sample mean of the auxiliary variate. The bias and the mean square error of $\hat{\bar{Y}}_R$ to the first degree of approximation is given below:

$$Bias(\hat{\bar{y}}_{R)} \cong \frac{1-f}{n}\bar{Y}\left(C_x^2 - 2C_xC_y\rho\right) \qquad (1.2)$$

$$MSE\,(\hat{\bar{y}}_{R)} \cong \frac{1-f}{n}\bar{Y}^2\left(C_y^2 + C_x^2 - 2C_xC_y\rho\right) \qquad (1.3)$$

Where, C_x, C_y are the co-efficient of variation and ρ is the co-efficient of correlation.

2. Yan and Tian (2010) [17] Estimator:

Yan and Tian (2010) [17] defined two modified ratio estimators using the linear combinations of Co-efficient of skewness and Co-efficient of kurtosis as

$$\hat{\bar{Y}}_1 = \bar{y}\left[\frac{\bar{X}\beta_2+\beta_1}{\bar{x}\beta_2+\beta_1}\right] \tag{2.1}$$

$$\hat{\bar{Y}}_2 = \bar{y}\left[\frac{\bar{X}\beta_1+\beta_2}{\bar{x}\beta_1+\beta_2}\right] \tag{2.2}$$

Where, $\beta_1 = \frac{N\sum_{i=1}^{N}(X_i-\bar{X})^3}{(N-1)(N-2)S^3}$ is Co-efficient of Skewness of auxiliary variable and $\beta_2 = \frac{N(N+1)\sum_{i=1}^{N}(X_i-\bar{X})^4}{(N-1)(N-2)S^3} - \frac{3(N-1)^2}{(N-2)(N-3)}$ is Co-efficient of kurtosis. To the first degree of approximation the bias and mean square error are given below:

$$Bias\left(\hat{\bar{Y}}_1\right) = \frac{(1-f)}{n}\bar{Y}(\theta_1^2 C_x^2 - \theta_1 C_x C_y \rho) \tag{2.3}$$

$$MSE\left(\hat{\bar{Y}}_1\right) = \frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_1^2 C_x^2 - 2\theta_1 C_x C_y \rho) \tag{2.4}$$

Where, $\theta_1 = \frac{\bar{X}\beta_2}{\bar{X}\beta_2+\beta_1}$

and

$$Bias\left(\hat{\bar{Y}}_2\right) = \frac{(1-f)}{n}\bar{Y}(\theta_2^2 C_x^2 - \theta_2 C_x C_y \rho) \tag{2.5}$$

$$MSE\left(\hat{\bar{Y}}_2\right) = \frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_2^2 C_x^2 - 2\theta_2 C_x C_y \rho) \tag{2.6}$$

Where, $\theta_2 = \frac{\bar{X}\beta_1}{\bar{X}\beta_1+\beta_2}$

3. The suggested adapted estimator:

We propose the new adopted or modified ratio estimator using the linear combination of Co-efficient of Kurtosis and Tri-Mean (TM) of the auxiliary variable. This measure is the weighted average of the population median and two quartiles. For more detailed properties of tri-mean see Wang *et al* (2007) [16].

$$\hat{\bar{Y}}_{p1} = \bar{y}\left[\frac{\bar{X}\beta_2+TM}{\bar{x}\beta_2+TM}\right], \tag{3.1}$$

Where, $\beta_2 = \frac{N(N+1)\sum_{i=1}^{N}(X_i-\bar{X})^4}{(N-1)(N-2)S^3} - \frac{3(N-1)^2}{(N-2)(N-3)}$, Co-efficient of kurtosis and (TM) which is the weighted average of the population median and two quartiles (Q1 and Q3) and is defined as:

TM= $(Q_1+Q_2+Q_3)/3$,

The bias of $\hat{\bar{Y}}_{p1}$ upto the first degree of approximation is given by;

$$B\left(\hat{\bar{Y}}_{p1}\right) = \frac{(1-f)}{n}\bar{Y}(\theta_{p1}^2 C_x^2 - \theta_{p1} C_x C_y \rho), \tag{3.2}$$

MSE of the above estimator can be found using Taylor series method as:

$$MSE\left(\hat{\bar{Y}}_{p1}\right) = \frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_{p1}^2 C_x^2 - 2\theta_{p1} C_x C_y \rho); \tag{3.3}$$

Where, $\theta_{p1} = \frac{\bar{X}\beta_2}{\bar{X}\beta_2+TM}$

4. Comparison of Efficiency

We compare the efficiencies for which the estimator given in (3.1) $\hat{\bar{Y}}_{p1}$ is more efficient than the existing estimator;

$$MSE\left(\hat{\bar{Y}}_{p1}\right) < MSE\left(\hat{\bar{Y}}_i\right) \text{ if } \rho < \frac{(\theta_{p1}+\theta_i)}{2}\frac{C_x}{C_y}\ ;\ 1,2\ (Yan\ and\ Tian\ (2010) \tag{4.1}$$

5. Numerical illustration

Data from reference book Murthy, M N (1967) [8] page 228 is utilized in which fixed capital is denoted by X (auxiliary variable) and output of 80 factories are denoted by Y (study variable) have been utilized for the purpose. For the assessment of the performance of the proposed estimator with that of the Yan and Tian (2010) [17] estimators. The population parameters are given in Table 1 and Constants, Bias and MSE's of estimators are given in Table 2 below:

Table 1: Parameters and various constants of population are

N	n	$\bar{Y}$	$\bar{X}$	ρ	S_y	C_y	S_x	C_x	β_2	β_1	T^M
80	20	51.8264	11.2646	0.9413	18.3569	0.3542	8.4542	0.750	2.866	1.05	9.318

Table 2: Constants, Bias and MSE's of estimators

Estimator	Constants	Bias	MSE
$\hat{\bar{Y}}_1$ Yan and Tian (2010) [17]	0.968	0.554	881.97
$\hat{\bar{Y}}_2$ Yan and Tian (2010) [17]	0.804	0.316	539.35
$\hat{\bar{Y}}_{p1}$ (Proposed Estimator)	0.787	0.305	415.27

% Relative Efficiency= MSE (Existing)/MSE (Proposed) * 100= 129.88%

It is evident from the Table.2 that the proposed estimator has smallest bias among the three as well as MSE value is less as compared to the Yan and Tian (2010) [17] estimators. We also calculate the value of the condition given in (4.1) $\rho < \frac{(\theta_m+\theta_i)}{2}\frac{C_x}{C_y}\ ;\ 1,2$ .Therefore the above condition is very much satisfied in case of modified estimator.

5. Conclusions

A new linear combination of Co-efficient of Kurtosis and Quartile deviation of the auxiliary variable has been adopted and a modified estimator has been proposed. Utilizing the numerical illustration it has also been demonstrated that the estimator produces smallest bias as well as MSE value as compared to the Yan and Tian (2010) [17] estimators.

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