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A review studies of Laplace's equations

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Abstract
The paper presents an efficiency study of the Laplace's equation. Thereby, Laplace's equation is just a belong of partial differential equation (PDE). Currently, the main method for solving the Laplace's equations is the finite element method (FEM) together with boundary element method (BEM). Moreover, in this manuscript, we are analyzing a plate of aluminum type alloy 1050 with Laplace's equation. The study with Laplace's equation was realize with Comsol Multiphysics software.

Keywords: Function, Equation, Laplace, Plate, Aluminum

1. Introduction

In mathematical analysis, a partial differential equation is an equation that imposes relationships between the various partial derivatives of multivariable function. Thus, in specialized books, the partial differential equation is abbreviated as PDE, [1].
The partial equation has the following types of equations:

- Lagrange's equation
- Poisson's equation
- Heat conduction equation
- Wave equation.

In this case, Laplace's equation is the simplest and most basic example of another types of partial differential equations. Anyway, the Laplace's equation is a linear homogeneity equation, [2].
Lagrange's equations are ubiquitous in mathematically oriented scientific fields such as economy, engineering and physics, as shown in Fig 1 below.

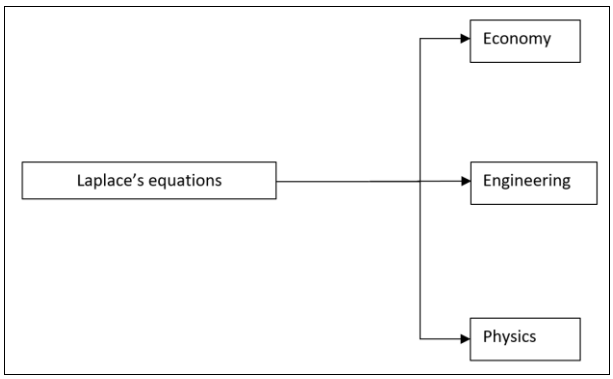

Fig 1: Lagrange's equation in mathematically oriented scientific fields

For example, they are fundamental to the modern scientific understanding of macroeconomics, sound, heat, electrostatics, etc.
The main stages of solving the Lagrange equations with finite element method are well clarified, as shown in Fig 2 below.

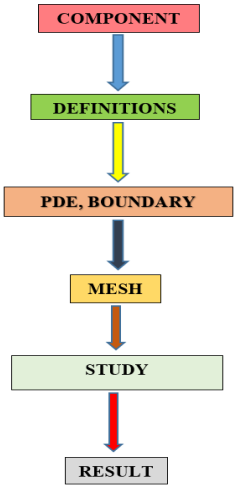

Fig 2: Steps for finite element study

In this paper, the analyzed sheet (thickness: $3 \cdot 10^{-3}$ m) is made of aluminum type alloy 1050, [3].
In the tables below shows mechanical and phisics propieties of aluminium alloy 1050.

a) Physical properties for aluminum Alloy 1050

Table 1: Physical properties of aluminum alloy 1050

Designation	Value	Unit
Density	2.72	kg/m^3
Melting point	923.15	K
Electrical resistivity	$0.283 \cdot 10^{-6}$	Ω·m
Thermal conductivity	222	W/m·K
Thermal expansion	$24 \cdot 10^{-6}$	1/K

b) Mechanical properties for aluminum Alloy 1050

Table 2: Mechanical properties of aluminum alloy 1050

Designation	Value	Unit
Proof stress	87	MPa
Tensile strength	110	MPa
Hardness Brinell	35	HB
Elongation A	12	%

2. Study of Lagrange's equations

The role of partial differential equations (PDEs) is that they describe the change of a system rather than its state in time and space. However, the Laplace's equation is available as a domain feature node for all the partial differential equations interfaces.
We consider a function ''u'' depends on two variables, x and y. Then the function u = f(x,y), [4].
Because, partial differential equations use such changes in more than one independent variable.
Further, we present some suggestive models about partial differential equations
The relationship for the coefficient from PDF interface is:

$$a \cdot u + \beta \cdot \nabla u + \nabla \cdot (\gamma - \alpha \cdot u - c\nabla u) + d_a \frac{\partial u}{\partial t} + e_a \frac{\partial^2 u}{\partial t^2} = f \quad (1)$$

Where:
- a – absorbtion coefficient
  $a = -1\ m^{-2}$
- β – convection coefficient
  $\beta_x = 0\ m^{-1}$ (x axis)
  $\beta_y = 0\ m^{-1}$ (y axis)
- γ – conservative flux source
  $\gamma_x = 0$ (x axis)
  $\gamma_y = 0$ (y axis)
- α – conservative flux convention coefficient
  $\alpha_x = 0$ (x axis)
  $\alpha_y = 0$ (y axis)
- c – diffusion coefficient for isotropic material (because isotropic material have properties which are independent of the direction of examination, x or y direction)
  c = 1
- d_a – damping
  $d_a = 1\ s \cdot m^{-2}$
- e_a - mass coefficient
  $e_a = 0\ s^2 \cdot m^{-2}$
- f – source term
  $f = 1\ m^{-1}$
- t – time
  t = 4 s

Of which some terms in formula 1 are scalars or vectors:
- Vectors: α, β and γ.
- Scalars: a, c, d_a and e_a.

The relation 1 is in the computational domain (Ω).
But symbol ∇ is the gradient operator, which has is the form:

$$\nabla = \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right] \quad (2)$$

Anyway, the Laplace operator (Δ) is dependent of the gradient operators:

$$\Delta = \nabla \cdot \nabla = \nabla^2 \quad (3)$$

Lapace's equation utilizate in mathematics is:
- Compact notation:

$$(\nabla u) \cdot u = 0 \quad (4)$$

- Component notation in 2D:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = u_{xx} + u_{yy} = 0 \quad (5)$$

Two boundary conditions for Laplace's equations are:
- Dirichelet boundary conditions
- Neumann boundary conditions

In our article, we only use Dirichelet boundary conditions.
In a partial differential equation (PDE), the relation of the Dirichelet boundary conditions are:

$$\nabla^2 u + u = 0 \quad (6)$$

In particular case, the equation of Dirichelet boundary conditions, are:

$$u = r \quad (7)$$

Where r is boundary term.
In the table below shows four boudary terms with their values used for the studied plate, [5].

Table 3: Boundary terms for the plate used

Boundary terms	Values
r_1	230
r_2	220
r_3	240
r_4	250

A plate of aluminum alloy 1050 has the following dimensions:

- Length: 1m.
- Width: 0.5 m.

Presentation of a plate with its dimensions, as in Fig 3.

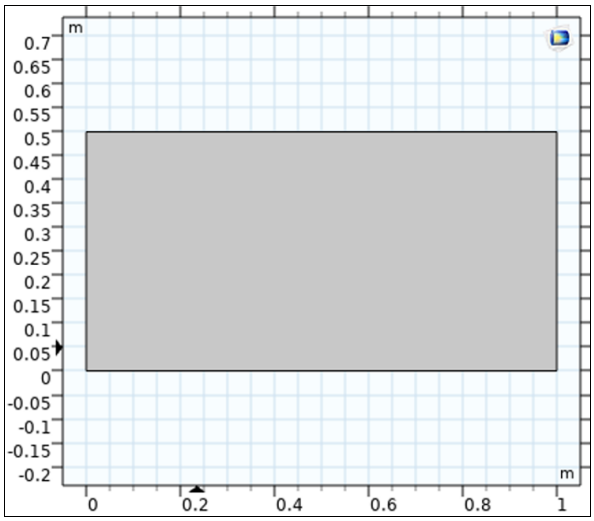

Fig 3: Plate of aluminum

The surface of the plate is dependent on the variable u, as in Fig 4.

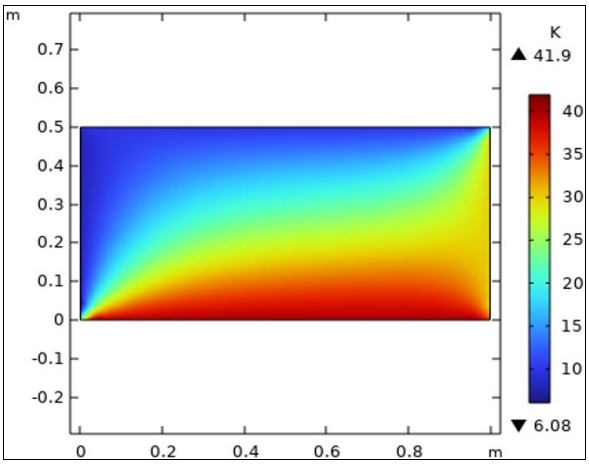

Fig 4: Surface – Dependent variable u(K). Simulation

The values of x belong to the length of the rectangle, [6].
The value of u increases with the value of x, then exponentially, as in Fig 5.

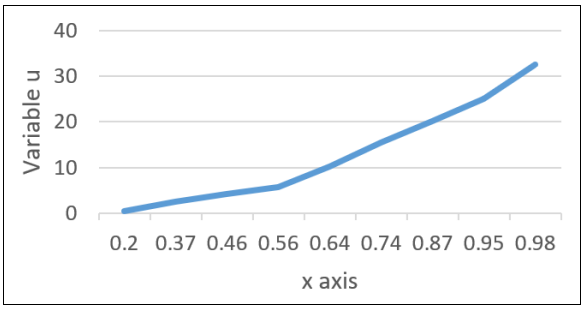

Fig 5: Graphic of variable u

3. Conclusions

The aluminum model, which is applied in the article, can be used for students in mathematical analysis classes in universities.
The plate of aluminum alloy 1050 is used for: food industry containers, pyrotechnic powder, lamp reflectors, cable sheating, architectural flashing, etc.
Thus, students can use more complex 2D models. This model can be developed for other materials: iron, steel, wood, cooper, etc.
In the future, we want to use Lagrange's equations for the study of 3D bodies.

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