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Thermodynamics of Casson Hybrid Nanofluid Flow Over a Melting Riga Plate in the Presence of Heat Source or Sink

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Abstract
This study emphasizes the effect of the flow rate, hybrid nanoparticles and thermal radiation on the magnetized mixed steady Casson nanofluid flow along the tailored size and shape of space shuttles and the nozzles of rocket engines. The similarity transformation was employed to reduce the difficulty of the governing equations of the problem to ordinary non-linear differential equations. The resulting governing equations are solved using Newton's finite difference method with Maple 18.0 software. The effect of dimensionless parameters such as Hartman numbers, Prandtl number, Magnetic parameter and Eckert on hybrid nanofluid flow, and mass transfer is considered. It was found that an increase in the value of the Harman number increases the hybrid nanofluid flow. The application of this work can be found in space technology, electromagnetic crucible system and spacecraft.

Keywords: Nanotechnology, Hybrid Nanofluid, Harman Number, Eckert Number, Prandtl Number, Cosmic Radiation

1. Introduction

Nanotechnology encompasses the changing and manipulating of the properties of matter at an atomic scale where conventional science such as physics and chemistry breaks down to confer new materials or devices with excellent performance properties that are far beyond those predicted by traditional approaches. For instance, Meador [1] explained that quantum confinement nanoscale semiconductor particle, quantum dots, gives rise to novel optical behaviour making it possible to tune the colour of their fluorescence simply by changing their diameter. Nanoscale texturing of surfaces can allow for control of adhesion properties leading to biomimetic (Gecko-foot) self-healing adhesives and self-cleaning surfaces. The unusual combination of superior mechanical properties, electrical and thermal conductivity and electronic properties of carbon-based nanostructured materials can enable the development of lightweight, multifunctional structures that will revolutionize the design of future aerospace systems. In regard to this [2, 3] showed that researchers are trying to utilize nanofluids as a way to increase the efficiency in solar energy applications. Metallic particles are becoming popular because of the plasmonic characteristics that allow for more absorption. Meanwhile, only homogenous nanofluids for solar applications have been considered. This article proposes the use of a hybrid nanofluid containing multiple types of nanoparticles with water as the base fluid exposed. However, Ungar and Erickson [4] narrated that the addition of metallic nanoparticles to a base heat transfer fluid can substantially increase its thermal conductivity. These nanofluids have been shown to have advantages in some heat transport systems. Their thermal properties allow the system's volumetric flow rate to be reduced, thus reducing the required pumping power. Nanofluids have been suggested as working fluids in spacecraft Active Thermal Control Systems (ATCSs). However, spacecraft ATCSs are unique in that they have stringent temperature control requirements and use specialized heat transfer devices. Recently Ahmadi et.al [5] reviewed the utilization of hybrid nanofluids in solar energy applications. It was shown that the enhanced thermophysical and rheological properties make them more appropriate for solar energy systems. Hafeez et.al. [6] considered the simulation of hybridized nanofluids flowing and heat transfer enhancement via 3-D vertical heated plate using finite element technique. Also, the effect of hybrid nanofluid on the heat transfer performance of parabolic trough solar collector receivers was investigated by Ekiciler et. al [7]. Center [8] wrote about the application of nanotechnology in space developments and systems. Moreover, Shutaywi and Shah [9] carried out mathematical Modeling and numerical simulation for nanofluid flow with entropy optimization. Singh [10] worked on the development of a unique multi-layer perceptron neural architecture and mathematical model for predicting the thermal conductivity of nanofluids with distilled water based using experimental data. Alghamdi et.al. [11] investigated the boundary layer stagnation point flow of the Casson hybrid nanofluid over an unsteady stretching surface. Gorla [12] explored the heat Source/Sink effects on a hybrid nanofluid-filled porous cavity. Usman et.al. [13] examined the Cu-Al2O3/Water hybrid nanofluid through a permeable surface in the presence of nonlinear radiation and variable thermal conductivity. Farooq et.al [14] carried out a study on melting heat transport and nanofluid in a nozzle of a liquid rocket engine with entropy generation. Also, Wang et.al [15] considered the thermal Distribution for Darcy–Forchheimer flow of Maxwell Sutterby nanofluids over a radiated extending surface. However, Nagendra *et al* [16] considered the of numerical study the hydromagnetic flow of heat and mass transfer in a nano Williamson fluid past a vertical plate with thermal and momentum slip effects. Eid and Makinde [17] explained the rheology of the combined impact of solar radiation, chemical reaction, Joule heating, viscous dissipation, and magnetic field on the flow of an electrically conducting nanofluid over a convectively heated stretching sheet embedded in a saturated porous medium. Adeshina *et al*. [18] studied the dynamics of heat transport of Casson nanofluid flow-induced Riga plate by stretching in the presence of a porous medium. El-Zahar [19] examined the unsteady MHD mixed convection flow of non-Newtonian Casson hybrid nanofluid in the stagnation zone of a sphere spinning impulsively. Hammachukiattikul *et al.* [20] considered an analytical study on sodium alginate-based hybrid nanofluid flow through a shrinking/stretching sheet with radiation, heat source and inclined Lorentz force effects. Asghar *et al*. [21] investigated the effect of thermal radiation on the three-dimensional magnetized rotating flow of a hybrid nanofluid. Ahmadinejad *et al*. [22] examined thermo-physical properties and thermal performance of Al2 O3 and CuO nanoparticles in water and ethylene glycol-based fluids. Samrat *et al*. [23] studied the impact of thermal radiation on an unsteady Casson nanofluid flow over a stretching surface and concluded that the nanoparticle plays an important role in enriching the effectiveness of the convection heat transfer of nanofluids. [24, 25] narrated the effect of viscous dissipation and thermal radiation on heat transfer over a stretching sheet through a porous medium. Mondal *et al*. [26] studied the unsteady MHD three-dimensional Casson nanofluid flow over a porous linear stretching sheet with slip conditions. Khan *et al* [27] An exact solution of a Casson fluid flow induced by dust particles with hybrid nanofluid over a stretching sheet subject to Lorentz forces. However, there is no unique solution presented, and then the stability of the solution was evaluated. The originality of this study is the investigation of continuous thermal solar radiation, Riga plate and heat source on the thermal transfer rate of the hybrid nanofluid flow along a stretching porous. The study was motivated by [28-31]. Hybrid nanofluid models were used as explanations for improved thermal and electrical conductivity, thermophoresis, and other thermos physical properties of the flow system. In the emergent area of nanotechnology, hybrid nanofluids have been one of the noticeable achievements of scientists and researchers, it is an open area of research due to its several applications in drag and heat dissipation technology such as solar cell, nanographene batteries, nanomedicine, nanostructured ceramic coatings, petroleum refining, biological filtration, photocatalytic water purification, nano-bioengineering of enzymes, atomic system and biofuel etc. where drag and heat dissipation are unavoidable. Meanwhile, the magnetic effect and heat dissipation reduction mechanism has been the area of emphasis because of their application in the prevention of mechanical energy loss. The reduction of drag and heat dissipation may create energy conservation, reduction of processing time, and thermal rating improvement making equipment more durable. The addition of nanofluid will boost the thermal conductivity of the medium.

2. Mathematical formulation

Following the work of Kumar *et al.* [28], the hybrid nanofluid flow over a stretching porous medium in a Cartesian coordinates system (x and y) is considered for steady flow processes. The space consists of a highly permeable stretching porous medium packed with Casson hybrid nanofluid consisting of Cu and $CoFe_2O_3$ in ethyl glycol ($C_2H_6O_2$) as a base fluid and Darcy's law is obeyed. As discussed by Ayub *et al.* [29], the Riga plate is embedded at the boundary layer along the x-direction to supply the Lorentz force to reduce hybrid nanofluid friction and pressure. Also, the modified magnetic field actuator also aids in the delay of boundary layer separation and the reduction of turbulence impacts. The hybrid nanofluid fluid velocity components at (x, y) directions are denoted by (u, v) respectively, where the y-axis is orthogonal to the surface and x-axis is along the stretching porous medium, and the stretching velocity is represented by. As stated by Adeshina *et al*. [30], the Riga plate with volume density of the Lorentz force was defined as: where is the current density, M is the magnetization in the magnet, s is the width of the magnet and the electrodes. However, the consumption of the reactants becomes negligible due to the addition of the hybrid nanoparticles hence the concentration equation is neglected. Meanwhile, the distribution of the hybrid nanoparticles into the base fluid is presumed to be in a state of thermal equilibrium. The single-phase model for this problem omits the cluster and sedimentation effects which reflects the stability of the hybrid nanofluid (Mohamed and Makinde [34]. The flow geometry is shown in Fig 1.

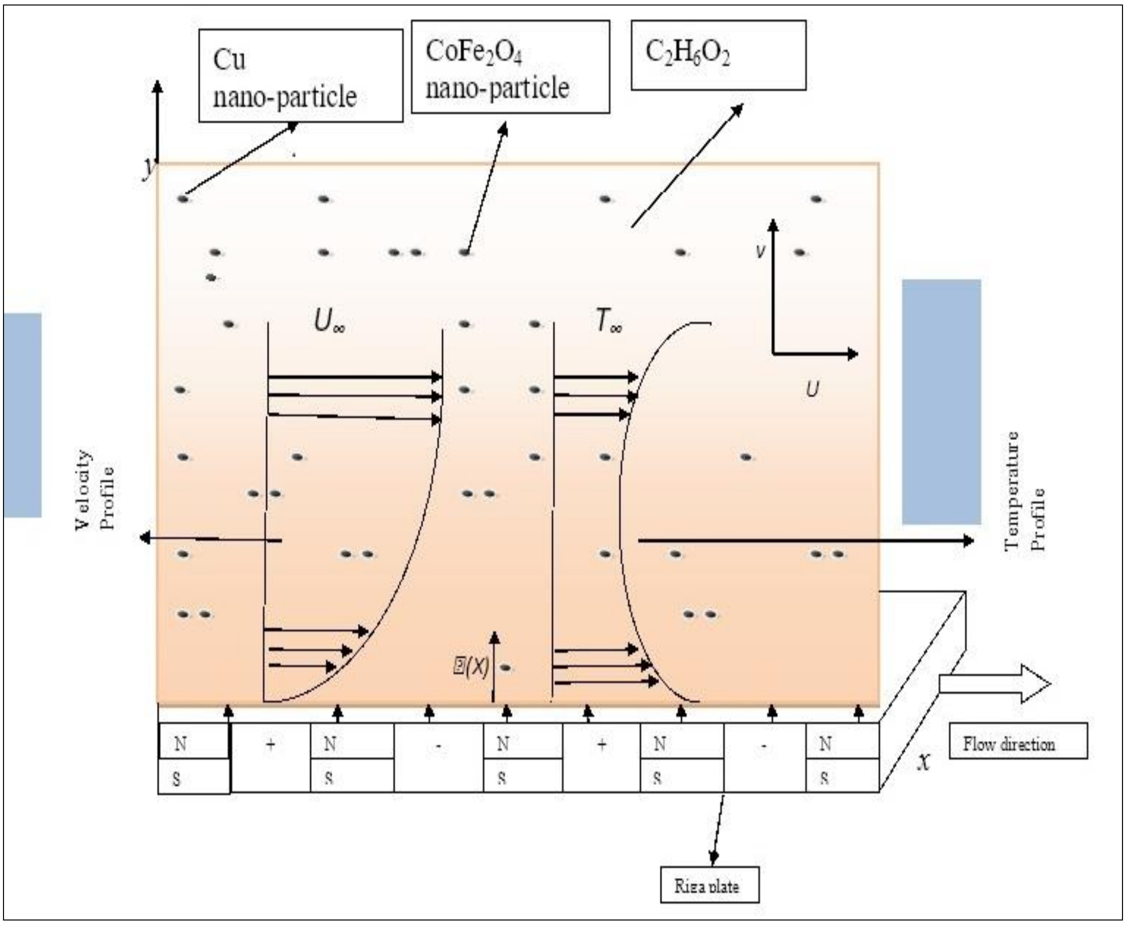

Fig 1: Flow model of hybrid nanofluid in a stretching porous medium with Riga plate

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho_{hnf}}\left(1+\frac{1}{\beta}\right)\frac{\partial}{\partial y}\left[\mu(T)\frac{\partial u}{\partial y}\right]+\frac{\pi j_0 M}{8\rho_{hnf}}e^{-\frac{\pi y}{s}}-\frac{\mu(T)}{\rho_{hnf}(kp)_0}\left(1+\frac{1}{\beta}\right)u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{(\rho c_p)_{hnf}}\frac{\partial}{\partial y}\left[K(T)\frac{\partial T}{\partial y}\right]+\left(1+\frac{1}{\beta}\right)\frac{\mu(T)}{(\rho c_p)_{hnf}}\left(\frac{\partial u}{\partial y}\right)^2+\left(1+\frac{1}{\beta}\right)\frac{\mu(T)}{(\rho c_p)_{hnf}(kp)_0}u^2-\frac{1}{(\rho c_p)_{hnf}}\frac{\partial q_r}{\partial y}+q''' \tag{3}$$

$$\left.\begin{array}{l} u=u_w+Lv\frac{\partial u}{\partial y}, v=-v_0, T=T_w, y=0, \\ u\rightarrow 0=, T\rightarrow T_\infty, as y\rightarrow\infty \end{array}\right\} \tag{4}$$

The suitable non-similarity transformations which were used to convert the equations governing the flow and their proper boundary conditions into dimensionless forms:

$$\left.\begin{array}{l} \eta=y\left(\frac{a}{v_\infty}\right)^{\frac{1}{2}} \\ u=axf'(\eta) \\ v=-(av_\infty)^{\frac{1}{2}}f(\eta) \\ \theta(\eta)=\frac{T-T_\infty}{T_m-T_\infty} \end{array}\right\} \tag{5}$$

$$\left.\begin{array}{l} \mu(T)=\mu_{hnf}[1+\varepsilon_1(T-T_\infty)]=\mu_{hnf}(1+\varepsilon_2-\varepsilon_2\theta) \\ k(T)=k_{hnf}[1+\varepsilon_3(T-T_m)]=k_{hnf}(1+\varepsilon_4\theta) \end{array}\right\} \tag{6}$$

The heat source parameter or (sink) q'' can be defined using as:

$$q'''=a[Q(T-T_\infty)+Q^*(T_m-T_\infty)e^{-\eta}] \tag{7}$$

Where Q>0, $Q^*>0$ is the heat generation while Q<0, $Q^*<0$ is the heat absorption

Consider the heat flux term represent the thermal radiation;

Following [33] and applying Rosseland approximations for thermal radiation and stratifying to an optically thick medium, the radiation heat flux given by (Raptis 1998), Brewester (1972), and Sparrow (1978)

$$q_r = -\frac{4\sigma*}{3k^*}\frac{\partial T^4}{\partial y} = -\frac{16T^3{}_\infty\sigma*}{3k^*}\frac{\partial T}{\partial y} \tag{8}$$

Where $k^*$and σ^* are the mean absorption coefficient. and Stefan-Boltzmann constant, respectively

$$\frac{1}{(\rho c_p)_{hnf}}\frac{\partial q_r}{\partial y} \tag{9}$$

Where,

$$q_r = -\frac{4\sigma*}{3k^*}\frac{\partial T^4}{\partial y} \tag{10}$$

The term T^4in equation can be expanded using Taylor series such that:

$$T^4 = 4T_\infty^3 T - 3T_\infty^4$$

$$q_r = -\frac{4\sigma*}{3k^*}\left(4T_\infty^3\frac{\partial T}{\partial y}\right) = -\frac{16T^3{}_\infty\sigma*}{3\,k^*}\frac{\partial T}{\partial y} \tag{11}$$

Following [33], the density, dynamic viscosity, electrical conductivity and specific heat capacity at constant pressure of the hybrid nanofluid were given below respectively as:

$$\rho_{hnf} = \rho_f\left((1-\phi_2)\left(1-\phi_1+\phi_2\left(\frac{\rho_{s1}}{\rho_f}\right)\right)+\phi_2\rho_{s2}\right)$$

$$\mu_{hnf} = \frac{\mu_f}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}}\frac{\mu_{hnf}}{\rho_{hnf}} = \frac{\mu_f}{\rho_f\left((1-\phi_2)\left(1-\phi_1+\phi_2\left(\frac{\rho_{s1}}{\rho_f}\right)\right)+\phi_2\rho_{s2}(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}\right)}$$

$$k_{hnf} = k_f * \frac{k_{s1}+(m-1)k_f-(m-1)\varphi_1(k_f-k_{s1})}{k_{s1}+(m-1)k_f+\varphi_2(k_f-k_{s1})} *$$

$$\left(\frac{k_{s2}+k_f\,(m-1)\left(\frac{k_{s1}+(m-1)k_f-(m-1)\varphi_1(k_f-k_{s1})}{k_{s1}+(m-1)k_f+\varphi_1(k_f-k_{s1})}-(m-1)\phi_2\left(k_f\left(\frac{k_{s1}+(m-1)k_f-(m-1)\varphi_1(k_f-k_{s1})}{k_{s1}+(m-1)k_f+\varphi_1(k_f-k_{s1})}\right)-k_{s2}\right)\right)}{k_{s2}+(m-1)k_f\left(\frac{k_{s1}+(m-1)k_f-(m-1)\varphi_1(k_f-k_{s1})}{k_{s1}+(m-1)k_f+\varphi_1(k_f-k_{s1})}\right)+\varphi_2\left(k_f\left(\frac{k_{s1}+(m-1)k_f-(m-1)\varphi_1(k_f-k_{s1})}{k_{s1}+(m-1)k_f+\varphi_1(k_f-k_{s1})}\right)-k_{s2}\right)}\right)$$

$$(\rho Cp)_{hnf} = (\rho Cp)_f\left((1-\phi_2)\left(1-\phi_1+\phi_1\frac{(\rho Cp)_{s1}}{(\rho Cp)_f}\right)+\phi_2(\rho Cp)_{s2}\right)$$

Let $\varepsilon_6 = (1-\phi_2)\left(1-\phi_1+\phi_2\left(\frac{\rho_{s1}}{\rho_f}\right)\right)+\phi_2\rho_{s2}(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}$

$$\varepsilon_7 = (1-\phi_2)\left(1-\phi_1+\phi_2\left(\frac{\rho_{s1}}{\rho_f}\right)\right)+\phi_2\rho_{s2}$$

$$\varepsilon_8 = \frac{k_{s1}+(m-1)k_f-(m-1)\varphi_1(k_f-k_{s1})}{k_{s1}+(m-1)k_f+\varphi_2(k_f-k_{s1})}*$$

$$\left(\frac{k_{s2}+k_f\,(m-1)\left(\frac{k_{s1}+(m-1)k_f-(m-1)\varphi_1(k_f-k_{s1})}{k_{s1}+(m-1)k_f+\varphi_1(k_f-k_{s1})}-(m-1)\phi_2\left(k_f\left(\frac{k_{s1}+(m-1)k_f-(m-1)\varphi_1(k_f-k_{s1})}{k_{s1}+(m-1)k_f+\varphi_1(k_f-k_{s1})}\right)-k_{s2}\right)\right)}{k_{s2}+(m-1)k_f\left(\frac{k_{s1}+(m-1)k_f-(m-1)\varphi_1(k_f-k_{s1})}{k_{s1}+(m-1)k_f+\varphi_1(k_f-k_{s1})}\right)+\varphi_2\left(k_f\left(\frac{k_{s1}+(m-1)k_f-(m-1)\varphi_1(k_f-k_{s1})}{k_{s1}+(m-1)k_f+\varphi_1(k_f-k_{s1})}\right)-k_{s2}\right)}\right)$$

$$\varepsilon_9 = (1-\phi_2)\left(1-\phi_1+\phi_1\frac{(\rho Cp)_{s1}}{(\rho Cp)_f}\right)+\phi_2(\rho Cp)_{s2}$$

Table 1: Thermo- physical properties of hybrid base fluid and nanoparticles

Physical properties	$C_2H_6O_2$	Cu	$CoFe_2O_3$	MoS_2	SiO_2
$\rho\left(\frac{kg}{m^3}\right)$	1113.5	4970	4970	2650	5060
$C_p\left(\frac{J}{kg\cdot k}\right)$	2430	385	700	730	397.746
$k\left(\frac{W}{m\cdot k}\right)$	0.253	401.0	3.7	1.5	34.5

$$\varepsilon_{10} = (1-\phi_1)^{2.5}(1-\phi_2)^{2.5}$$

$$\frac{1}{\varepsilon_6}(1+\varepsilon_2-\varepsilon_2\theta)\left(1+\frac{1}{\beta}\right)f''' + ff'' + \frac{1}{\varepsilon_7}Ze^{-P\eta}-(f')^2 -$$

$$\frac{\varepsilon_2}{\varepsilon_6}\left(1+\frac{1}{\beta}\right)\theta' f'' - \frac{1}{\varepsilon_6}P_p(1+\varepsilon_2-\varepsilon_2\theta)\left(1-\frac{1}{\beta}\right)f'(\eta)=0 \quad (12)$$

$$\frac{\varepsilon_8}{\varepsilon_7}(1+\varepsilon_4\theta)\theta'' + \frac{\varepsilon_8}{\varepsilon_7}\varepsilon_4(\theta')^2 + Ra\,\theta'' + \Pr f\theta' +$$

$$\Pr Ec\left(1+\frac{1}{\beta}\right)\frac{(1+\varepsilon_2-\varepsilon_2\theta)}{\varepsilon_9}(f'')^2 + Pp\Pr Ec\left(1+\frac{1}{\beta}\right)\frac{(1+\varepsilon_2-\varepsilon_2\theta)}{\varepsilon_6}f'^2 +$$

$$\Pr\left[Q\theta+Q^*e^{-\eta}\right] \quad (13)$$

Following Kumar *et al.* (2018), the corresponding boundary condition is given as:

$$\left.\begin{array}{l} f'(0)=1, \theta(0)=0, M_e(1+\varepsilon_4\theta(0))\theta'(0)+Pr\,f(0)=0 \\ f'(\infty)\to 0, \theta(\infty)\to 1\eta\to\infty \end{array}\right\} \quad (14)$$

Where the following are dimensionless number:

$Pr = \frac{\rho_f c_p v_\infty}{k_f}$ is the Prandtl number, $Ra = \frac{16\sigma * T^3_\infty}{3k^* k_f}$ is Radiation parameter

Ec = $\frac{a^2 x}{c_p(T_w-T_\infty)}$ is the Eckert number, $P_p = \frac{v_\infty}{a(kp)_0}$ is the Porosity term

$Z = \frac{\pi j_0 M v_\infty \rho_f}{8a^2 x \mu_f \rho_f}$ is the modified Hartman number.

3. Result and discussion

The section analyzes the effect of dimensionless parameters such as the Casson parameter, melting parameter (Me), Hartman number (*Z*), Porosity (Pp), Radiation parameter (Ra), variable viscosity, and parameter (P) on fluid velocity and thermal field. Fig 2 explained the relationship between fluid velocity and the melting parameter (Me). The increase in the value of Me causes rising the value of. This is due to the fact that an increase in Me leads to a boost in melting influence which consequently increases the hybrid nanofluid velocity, Adeosun *et al*. (2021). Fig 3 explained the effect of the Casson parameter. It was noticed that the velocity decreases as it increases. This is associated with the fact that for the greater values of the Casson parameter, the dynamic viscosity strength becomes stronger and resists the nanofluid flow. It is a known fact that increasing the Casson parameter decreases the yield stress of the Casson fluid, and increasing it indefinitely will make the fluid behave as a Newtonian fluid. It is obvious that fluid motion is slowed down in both directions due to an increase in the value of the Casson parameter due to the no-slip boundary condition, which means a decrease in the velocity profiles and leads to a decrease in the momentum boundary layer thickness.

This behaviour is observed in Nadeem *et al*. (2014) and Mukhopadhyay (2013) and Oyelakin *et al* (2017). It is also observed in Fig 4 that the velocity of the hybrid nanofluid is depreciated as parameter (P) increases. Also, Fig 5 shows that the increase in the effect of the temperature-dependent viscosity parameter, tends to increase the nanofluid velocity. This might occur because of an increase in temperature difference which consequently, reduces the dynamic viscosity of the fluid and enhances the nanofluid flow. This validated the work of Adeosun *et al.* (2021). Fig 6 explained the effects of the modified Hartman number (Z) on the velocity of the hybrid nanofluid. It is observed that as the value of Z rises, the velocity is elevated. This means that rising values of Z correspond to the higher intensity of the magnetic field strength mounted on the parallel plate which leads to the enhancement of the nanofluid flow over the plate by reducing the fluid friction. This validated the work of Ganesh *et al.* (2019) and Adeosun *et al*. (2021)

Fig 7 shows that rising values of porosity reduce the fluid flow in a stretching porous medium. This is because of an increase in porosity parameter associated with a reduction in flow permeability which results in the retardation of nanofluid flow. Fig 8 illustrated the effect of hybrid nanofluid concentration on fluid velocity. It was noticed that an increase in hybrid nanofluid concentration leads to a decrease in fluid velocity. This is because of an increase in dynamic viscosity as the concentration of the hybrid nanofluid is increasing. In Fig 9, it was noticed that the increase in hybrid nanofluid melting point caused decreases in the hybrid nanofluid temperature. This is because more energy will be released and consequently leads to a decrease in temperature. Fig 10 shows that an increase in the thermal radiation parameter (Ra) enhances the temperature of the hybrid nanofluid. Clearly, it is evident that Ra contributes to increasing temperature distribution because radiation enhances the internal energy of fluid and therefore, temperature rises. This validates the work of (Iqbal *et al*, 2017).

Fig 11 explained the effect of hybrid nanofluid concentration on the temperature of the hybrid nanofluid flow in a stretching porous medium. The increase in the value increases the temperature of the hybrid nanofluid because increasing concentration leads to an increase in the frequency of collision of the hybrid nanoparticles, therefore, increasing the temperature. This validates the experimental work of (Ooi *et al*,2022)

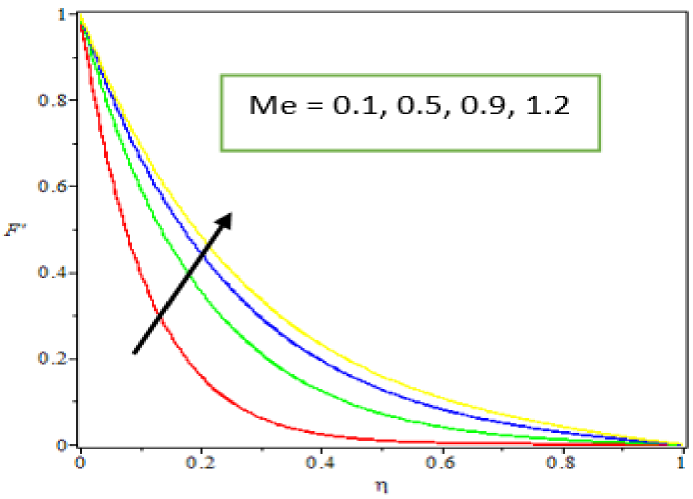

Fig 2: Influence Melting parameter Me on $F'(\eta)$

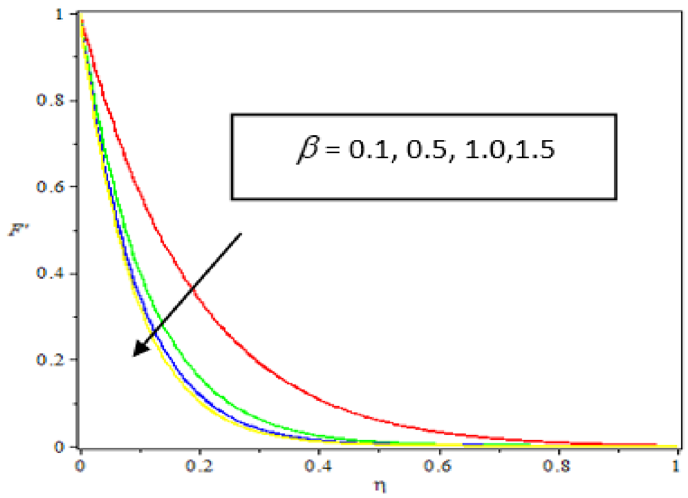

Fig 3: Influence of Casson parameter β on $F'(\eta)$

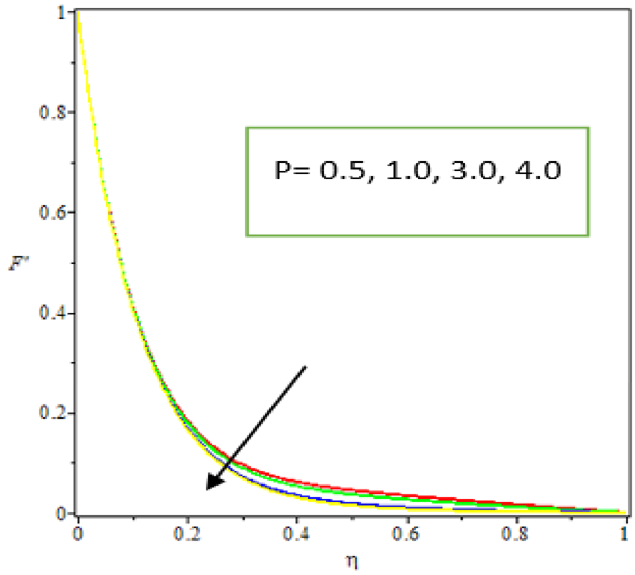

Fig 4: Influence of Parameter P on $F'(\eta)$

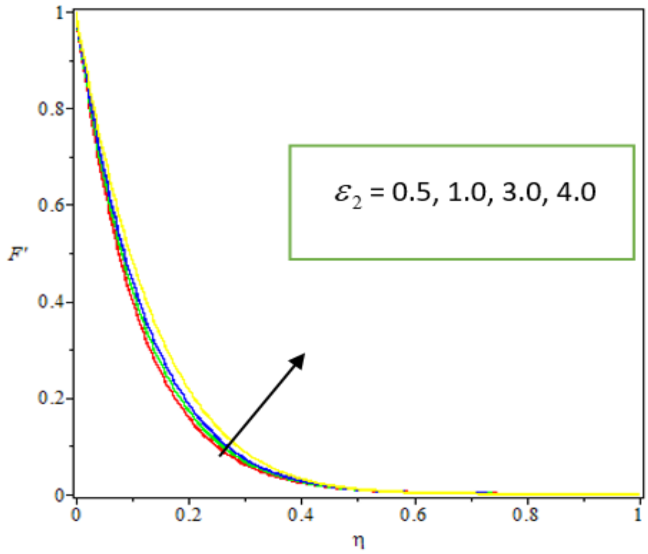

Fig 5: Influence of $\varepsilon_2$on $F'(\eta)$

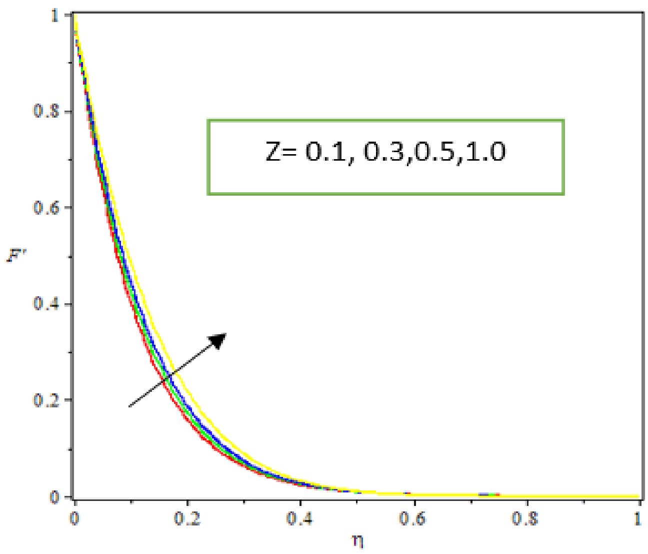

Fig 6: Influence of Hartman number Z on $F'(\eta)$

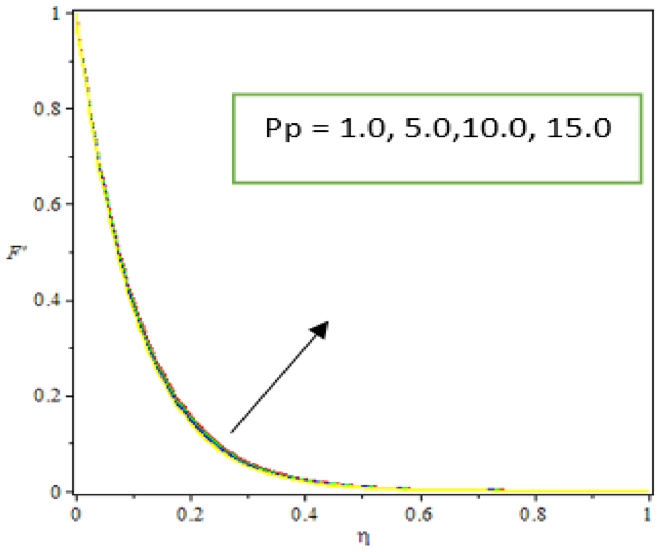

Fig 7: Influence of Porosity Pp on $F^{'}(\eta)$

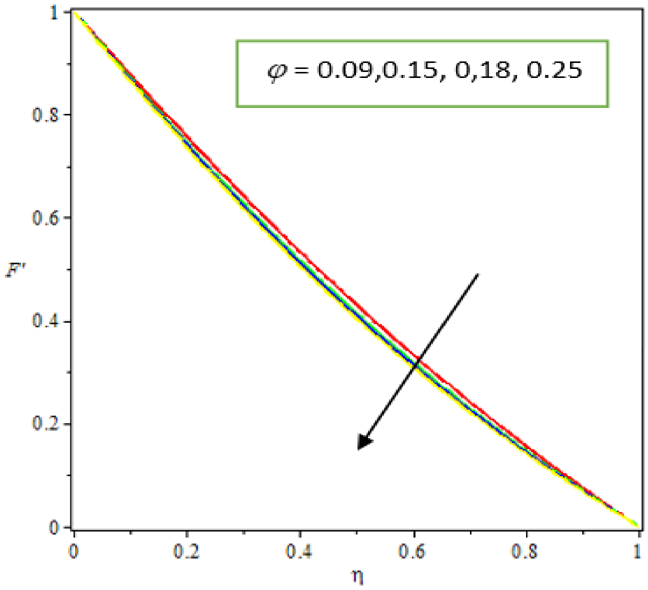

Fig 8: Influence ϕ on $F^{'}(\eta)$

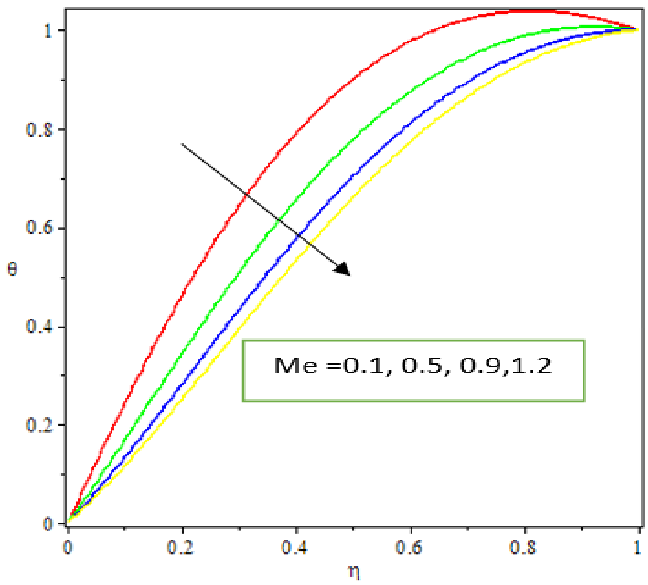

Fig 9: Influence of Me on $\theta(\eta)$

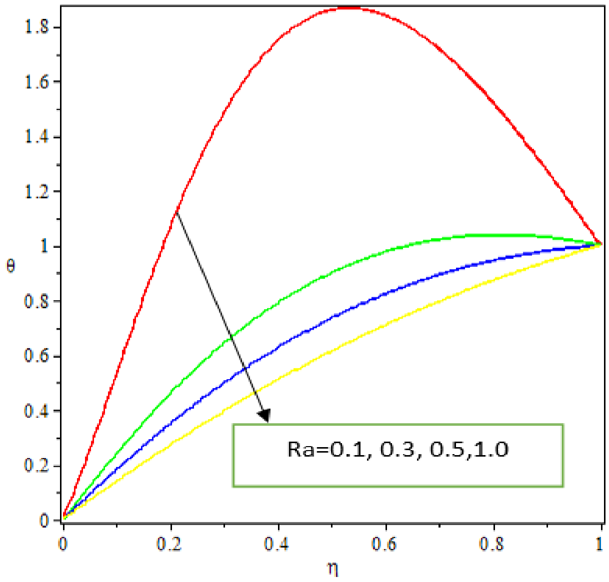

Fig 10: Influence of Ra on $\theta(\eta)$

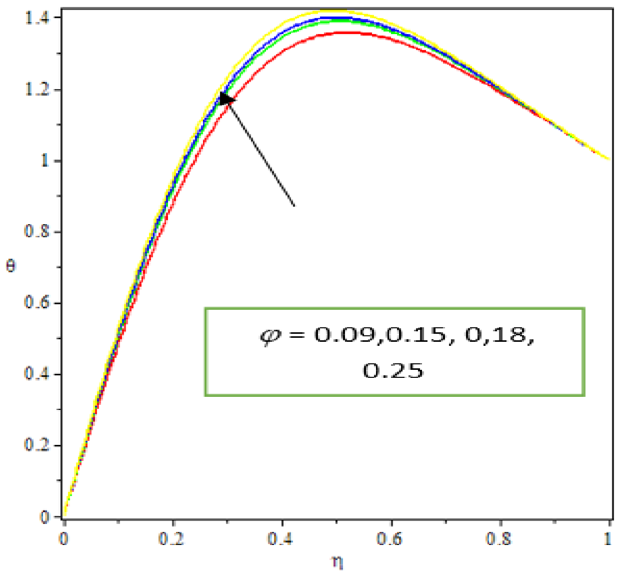

Fig 11: Influence of ϕon $\theta(\eta)$

4. Conclusion

The steady magnetohydrodynamics Casson hybrid nanofluid flow through a stretching porous medium was investigated with influence variable dynamic viscosity, thermal radiation, and Riga plate. Based on the results of this study, the following conclusions were deduced. A new mathematical model for the flow geometry of magnetohydrodynamics of Casson hybrid nanofluid for steady flow is established. Analysis of the transformed governing equations of thermodynamics of steady magnetohydrodynamics Casson hybrid nanofluid flow through a stretching porous medium with the similarity transformation via FDM showed that Riga plate, hybrid nanofluid concentration and thermal radiation enhance the temperature profile which results in to increase in the fluid flow. Some of the following important notes were arrived at:

1. The hybrid velocity is increasing as the value of the Hartman number is increasing.
2. The increase in hybrid nanofluid concentration reduces fluid flow.
3. Increase in hybrid nanofluid concentration enhance the fluid temperature.
4. Thermal radiation has a positive effect on the hybrid nanofluid temperature.
5. The increase in the value of the temperature-dependent dynamic viscosity term enhances fluid velocity.

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Conflicts of Interest: The authors declare that they have no competing interests.

Table 1: Nomenclature

(x, y)	Cartesian coordinate
u, v:	Fluid velocity in x and $y$direction
ρ_f:	Fluid density
ρ_{s1}:	Density of the Hybrid nanoparticle MoS_2
ρ_{s2}:	Density of the Hybrid nanoparticle SiO_2
μ_f:	Dynamic viscosity of fluid
β	Casson Parameter
$B_\circ$:	Applied magnetic induction
g:	Acceleration due to gravity
T:	Fluid temperature
T_∞:	Free stream temperature
$Q_\circ$:	Dimensional heat generation coefficient
σ:	Electrical conductivity of the surface temperature
σ_{hnf}:	Electrical conductivity of the Hybrid Nanofluid
σ_1:	Electrical conductivity of Hybrid Nanoparticle
σ_2:	Electrical conductivity of Hybrid Nanoparticle
β_T:	Thermal expansion Coefficient
U_w:	Reference velocity
v_{hnf}:	Kinematic viscosity hybrid nanofluid
f:	Velocity profile
η:	Similarity variable
θ:	Temperature
τ:	Particles heat capacity to fluid heat
ε_2:	Variable viscosity term
T_w:	Wall temperature
$\in_4$:	Variable thermal conductivity parameter
K_∞;	Constant thermal conductivity
A:	Heat source parameter
μ_∞:	Constant viscosity
ψ:	Stream function
M:	Magnetic Parameter
F_s:	Forchheimer parameter
G_r:	Thermal Grashof number
ϕ_1:	The volume fraction of nanoparticle solid MoS_2
ϕ_2:	The volume fraction of nanoparticle solid SiO_2
E_c:	Eckert number
P_r:	Prandtl number
∂q_r	Radiative heat flux

5. References

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